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OPTIMUM TUNED DAMPERS FOR RANDOMLY EXCITED DYNAMIC SYSTEMS

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UNIVERSITY OF ILLINOIS

TECHNICAL REPORT No. AFML-TR-67-217

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FOREWORD

This report was prepared by the Department of Aeronautical and Astronautical Engineering at the University of Illinois under USAF Contract No. F33615-67-C1190. This contract was initiated under Project No. 7351, "Metallic Materials," Task No. 735106, "Behavior of Metals." The work was administered under the direction of the Air Force Materials Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, Lt. G. H. Bruns, MAMD, acting as Project Engineer.

This research was conducted under the direction of Professor Y. K. Lin. This report covers work conducted from November 1, 1966 to April 30, 1967.

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The manuscript was released by the author May 1967 for publication.

This technical report has been reviewed and is approved.

W. J. Trapp

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ABSTRACT

The objective of this report is to present the effect of a tuned damper on a single degree-of-freedom system which is subjected to white noise excitation. The tuned damper itself consists of a mass connected to a visco-elastic link which, in turn, is connected to the primary system under consideration. The criterion used for tuning the damper is the minimization of the mean square response of the primary system. The tuned damper obtained by use of this criterion is compared to that obtained from another criterion requiring the peaks of the absolute value of the frequency response function to be of equal height.

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LIST OF SYMBOLS

E[ ]	expected value of bracketed random quantity
h	impulse response function
H	frequency response function
H^*	complex conjugate of H
i	$\sqrt{-1}$
$k_{1,2}$	spring constants
K	spectral density of a white noise
$m_{1,2}$	masses
M	mass ratio, m_2/m_1
R	autocorrelation function
t	time
X	random displacement
x	deterministic displacement
W	white noise excitation

GREEK SYMBOLS

η	loss factor of viscoelastic link
θ	angle
λ	pole in complex plane
Φ	spectral density

GREEK SYMBOLS (continued)

τ	time
ω	circular frequency

SUBSCRIPTS

$(\ )_{FF}$	refers to input
$(\ )_{XX}$	refers to output

1. INTRODUCTION

Reduction of the amplitude of a vibrating system through the use of various damping devices has received extensive study (see References 1, 4, 7, 8, and 9). A method of damping that is presently receiving some attention is that of using viscoelastic damper units (see References 3, 4, 5, and 10). This unit consists of a mass attached to a viscoelastic link which is, in turn, attached to the primary system under consideration.

Consider a primary system consisting of a spring with a spring rate k_1 which suspends a mass m_1 as shown in Figure 1. If a viscoelastic damper unit, idealized as a small mass connected to a spring with a complex modulus, is attached to the primary system, the absolute value of the frequency response function exhibits two peaks. The frequency response function is the complex ratio of the steady-state response of the system to a sinusoidal input and is a function of the excitation frequency. It has been suggested (see References 10, 11) that an optimum damper would damp the motion of the primary system such that the two peaks of the absolute value of the frequency response function would be of equal height.

This report outlines an investigation into another possible criterion for optimization of viscoelastic dampers. The same primary spring-mass system, to which is attached a viscoelastic damper unit, is considered. The system is excited by a special type of random excitation, a white noise. The criterion that is proposed for optimization of this system

is that the mean square value of the weakly stationary random response be a minimum. Although a white noise is considered, the results are applicable to any excitation which exhibits a broad band flat spectral density over a range of frequencies of practical interest. See Reference 12.

The method of residues for complex variables is employed in determining the mean square value of the weakly stationary response. Numerical results are presented for various combinations of the parameters involved and are compared with those corresponding to the criterion of equal peaks for the absolute value of the frequency response function. The numerical computations were carried out on a high speed computer.

2. ANALYSIS

As is stated in the introduction, the criterion that is proposed for optimization of the viscoelastic damper unit is to obtain a minimum mean square value of the weakly stationary response when the system is excited by a white noise. When this condition is satisfied, the damper unit will be referred to as tuned. Since the input is random, the response is also random. The springs and masses making up the primary system and the damper unit are considered to be deterministic.

If the system is excited by a weakly stationary random excitation, such as a white noise, the response of the system also becomes weakly stationary after a sufficient amount of time has passed so that the transient motion has died out. In this manner, the weakly stationary response is analogous to the steady state response in deterministic vibration theory. It may be called the steady state in the probabilistic sense. The time required for this condition to occur is dependent upon the amount of damping in the system. The greater the damping, the sooner the response becomes weakly stationary.

Two cases involving random excitation are considered. Figure 1 corresponds to the case of random excitation applied at mass m_1 of the system. Figure 2 corresponds to the case where the foundation is moved in a random fashion. The tuned dampers corresponding to these cases are compared to the tuned dampers corresponding to the criterion for equal peaks of the absolute value of the respective frequency response functions.

When the random input is weakly stationary, the autocorrelation function of the input is a function of the difference in the parametric values:

$$E[F(\tau_1)F(\tau_2)] = R_{FF}(\tau_1 - \tau_2) \tag{1}$$

The autocorrelation function can be expressed as the fourier transform of the spectral density; that is

$$R_{FF}(\tau_1 - \tau_2) = \int_{-\infty}^{\infty} \Phi_{FF}(\omega)\, e^{i\omega(\tau_1 - \tau_2)}\, d\omega \tag{2}$$

In order to compute the correlation function of the response, use is made of the following relationship between the random input and the random output

$$E[X(t_1)X(t_2)] = \int_0^{t_1} d\tau_1 \int_0^{t_2} d\tau_2\, E[F(\tau_1)F(\tau_2)]\, h(t_1 - \tau_1)h(t_2 - \tau_2) \tag{3}$$

It has been assumed that $X(t) = X(t) = 0$ at $t = 0$ with probability one. Substituting Equations (1) and (2) into (3) the correlation function of the response can be written:

$$E[X(t_1)X(t_2)] = \int_0^{t_1} d\tau_1 \int_0^{t_2} d\tau_2 \int_{-\infty}^{\infty} d\omega\, \Phi_{FF}(\omega)\, e^{i\omega(\tau_1 - \tau_2)} h(t_1 - \tau_1)h(t_2 - \tau_2) \tag{4}$$

Integrating first on τ_1 and τ_2:

$$E[X(t_1)X(t_2)] = \int_{-\infty}^{\infty} \Phi_{FF}(\omega)\, \mathcal{H}(\omega, t_1)\, \mathcal{H}^*(\omega, t_2)\, e^{i\omega(t_1 - t_2)}\, d\omega \tag{5}$$

where the following notation has been used:

$$\mathcal{H}(\omega,t) = \int_0^t h(u)e^{-i\omega u}\,du \tag{6}$$

The interchange of the order of integration in Equation (4) is permissible provided that the function $\mathcal{H}(\omega,t)$ is uniformly bounded in ω. This condition is always satisfied for systems with positive damping. It may be noted that the lower limit of the integral in Equation (6) can be extended to $-\infty$, since h(u) vanishes for negative u. Furthermore, if the upper limit t tends to infinity, then the right hand side of Equation (6) becomes the frequency response function, that is:

$$\mathcal{H}(\omega,\infty) = H(\omega)$$

Since only the mean square value of X(t) in the weakly stationary state will be considered, we let $t_1 = t_2 = t$ and let t tend to infinity in Equation (5). We obtain:

$$E[X^2(t)] = \int_{-\infty}^{\infty} |H(\omega)|^2 \, \Phi_{FF}(\omega)\,d\omega \tag{7}$$

Equation (7) describes the relationship between the weakly stationary mean square response of the system, the spectral density of the random input, and the absolute value squared of the frequency response function. The absolute value squared of the frequency response function prescribes the fraction of energy to be transmitted through the system at various frequencies.

Integration of Equation (7) can be performed if the spectral density of the excitation is known. The random excitation considered in this report is white noise excitation. For this excitation, the spectral density is a constant. The physical interpretation of a constant spectral density is that the energy content in the random forcing function is uniformly distributed over the entire frequency domain. Since this corresponds to an infinite mean energy, a white noise is physically impossible. However, if the absolute value of the frequency response function is sharply peaked near the natural frequencies of the system, and the actual input spectral density varies slowly in the neighborhood of the peaks, then the excitation can be treated as white noise while computing the second order properties of the response. The white noise excitation is then physically meaningful in the sense of being a good approximation to an actual spectral density for such computations.

Since the spectral density for a white noise is a constant, K, the expression for the mean square value of the weakly stationary response can be written:

$$E[X^2(t)] = K \int_{-\infty}^{\infty} \left| H(\omega) \right|^2 d\omega \tag{8}$$

The viscoelastic element in the damper unit is represented by a spring with a complex valued stiffness,

$k_2(1 + i\eta)$, where η is referred to as the loss factor. For the system shown in Figure 1, it can be shown that:

$$|H(\omega)|^2 = \frac{[\omega_2^2(1+i\eta) - \omega^2]}{m_1[\omega^4 - \omega_2^2\omega^2(1+M) - \omega_1^2\omega^2 + \omega_1^2\omega_2^2] + im_1[-\omega_2^2\omega^2\eta(1+M) + \eta\omega_1^2\omega_2^2]} \cdot \tag{9}$$

$$\frac{[\omega_2^2(1-i\eta) - \omega^2]}{m_1[\omega^4 - \omega_2^2\omega^2(1+M) - \omega_1^2\omega^2 + \omega_1^2\omega_2^2] - im_1[-\omega_2^2\omega^2\eta(1+M) + \eta\omega_1^2\omega_2^2]}$$

where:

$$\omega_2^2 = \frac{k_2}{m_2} \; ; \; \omega_1^2 = \frac{k_1}{m_1} \; ; \quad M = \frac{m_2}{m_1} \tag{10}$$

This frequency response function refers to the displacement of mass m_1. It is the response of this mass that is of primary importance since this mass represents the physical structure.

Equation (8) can be evaluated by the method of residues. The integrand, which is a function of the real variable ω, can be treated as a function of a complex variable λ. Since the method of residues is employed, the poles of the right hand side of Equation (9) must be located. Consider the following portion of the integrand (where ω has been replaced by λ):

$$\frac{\omega_2^2(1+i\eta) - \lambda^2}{m_1[\lambda^4 - \omega_2^2\lambda^2(1+M) - \omega_1^2\lambda^2 + \omega_1^2\omega_2^2] + im_1[-\omega_2^2\lambda^2\eta(1+M) + \eta\omega_1^2\omega_2^2]} \tag{11}$$

To locate the poles, the denominator is set equal to zero:

$$\lambda^4 - \lambda^2[\omega_2^2(1+M) + \omega_1^2 + i\eta\omega_2^2(1+M)] + \omega_1^2\omega_2^2(1+i\eta) = 0 \tag{12}$$

Solving for λ^2 :

$$\lambda^2 = \frac{1}{2}[\omega_2^2(1+M) + \omega_1^2 + i\eta\omega_2^2(1+M)] \pm \frac{1}{2}\left\{[\omega_2^2(1+M) + \omega_1^2 + i\eta\omega_2^2(1+M)]^2 - 4\omega_1^2\omega_2^2(1+i\eta)\right\}^{1/2} \tag{13}$$

This expression can be rewritten as follows:

$$\lambda^2 = \frac{\omega_2^2 A + \omega_1^2}{2} \pm \frac{\sqrt{D}}{2}\cos\left(\frac{\theta}{2}\right) + \frac{i}{2}[\eta\omega_2^2 A \pm \sqrt{D}\sin\left(\frac{\theta}{2}\right)] \tag{14}$$

where:

$$D = \sqrt{R^2 + I^2} \tag{15}$$

$$\theta = \tan^{-1}\frac{I}{R} \tag{16}$$

$$R = A^2\omega_2^4 + \omega_1^2\omega_2^2\ (2A-4) + \omega_1^4 - \eta^2 A^2\omega_2^4 \tag{17}$$

$$I = 2\eta A^2\omega_2^4 + \eta\omega_1^2\omega_2^2\ (2A-4) \tag{18}$$

$$A = 1 + M \tag{19}$$

Note that the principal value of θ is to be used in Equation (14). If the following substitutions are made,

$$E_1 = \frac{\omega_2^2 A + \omega_1^2}{2} + \frac{\sqrt{D}}{2}\cos\left(\frac{\theta}{2}\right) \tag{20}$$

$$E_2 = \frac{\omega_2^2 A + \omega_1^2}{2} - \frac{\sqrt{D}}{2}\cos\left(\frac{\theta}{2}\right) \tag{21}$$

$$F_1 = \frac{1}{2}\left[\eta\omega_2^2 A + \sqrt{D}\sin\left(\frac{\theta}{2}\right)\right] \tag{22}$$

$$F_2 = \frac{1}{2}\left[\eta\omega_2^2 A - \sqrt{D}\sin\left(\frac{\theta}{2}\right)\right] \tag{23}$$

then the expression for λ^2 can be further simplified to:

$$\lambda^2 = E_1 + i F_1 \tag{24}$$

$$\lambda^2 = E_2 + i F_2 \tag{25}$$

The poles of expression (11) are now given as follows:

$$\lambda_1 = \sqrt{G_1}[\cos(\frac{\theta^*}{2}) + i \sin(\frac{\theta^*}{2})] \tag{26}$$

$$\lambda_2 = -\sqrt{G_1}[\cos(\frac{\theta^*}{2}) + i \sin(\frac{\theta^*}{2})] \tag{27}$$

$$\lambda_3 = \sqrt{G_2}[\cos(\frac{\theta^{**}}{2}) + i \sin(\frac{\theta^{**}}{2})] \tag{28}$$

$$\lambda_4 = -\sqrt{G_2}[\cos(\frac{\theta^{**}}{2}) + i \sin(\frac{\theta^{**}}{2})] \tag{29}$$

where:

$$G_1 = \sqrt{E_1^2 + F_1^2} \tag{30}$$

$$\theta^* = \tan^{-1} F_1/E_1 \tag{31}$$

$$G_2 = \sqrt{E_2^2 + F_2^2} \tag{32}$$

$$\theta^{**} = \tan^{-1} F_2/E_2 \tag{33}$$

Again, only the principal values of θ^* and θ^{**} are used in Equations (26) thru (29). Or, more simply:

$$\lambda_1 = a + ib \tag{34}$$

$$\lambda_2 = -a - ib \tag{35}$$

$$\lambda_3 = u + iv \tag{36}$$

$$\lambda_4 = -u - iv \tag{37}$$

Returning to Equation (9) it is seen that the portion of the absolute value squared of the frequency response function that has not been considered is the complex conjugate of the portion that has just been considered. It can be shown that the poles of the portion yet to be considered are of the form:

$$\lambda_5 = a - ib \tag{38}$$

$$\lambda_6 = -a + ib \tag{39}$$

$$\lambda_7 = u - iv \tag{40}$$

$$\lambda_8 = -u + iv \tag{41}$$

If, for example, a, b, u, and v are all positive, one possible location of the eight values of λ is shown in Figure 3.

We are now ready to perform the integration shown in Equation (8). The limits on this integral are minus infinity and plus infinity. Replace the right hand side of Equation (8) by a contour integration and let the contour be as shown in Figure 3. Application of the residue theorem yields:

$$E[X^2(t)] = 2\pi\ Ki(R_1 + R_3 + R_6 + R_8) \tag{42}$$

where R_j is the residue at $\lambda = \lambda j$. These residues are readily evaluated as follows:

$$R_1 = \frac{\omega_2^4 + \omega_2^4 \eta^2 - 2\omega_2^2 \lambda_1^2 + \lambda_1^4}{m_1^2(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)(\lambda_1 - \lambda_4)\dots(\lambda_1 - \lambda_8)} \tag{43}$$

$$R_3 = \frac{\omega_2^4 + \omega_2^4 \eta^2 - 2\omega_2^2 \lambda_3^2 + \lambda_3^4}{m_1^2(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_4)\dots(\lambda_3 - \lambda_8)} \tag{44}$$

$$R_6 = \frac{\omega_2^4 + \omega_2^4 \eta^2 - 2\omega_2^2 \lambda_6^2 + \lambda_6^4}{m_1^2(\lambda_6 - \lambda_1)(\lambda_6 - \lambda_2)\dots(\lambda_6 - \lambda_5)(\lambda_6 - \lambda_7)(\lambda_6 - \lambda_8)} \tag{45}$$

$$R_8 = \frac{\omega_2^4 + \omega_2^4 \eta^2 - 2\omega_2^2 \lambda_8^2 + \lambda_8^4}{m_1^2(\lambda_8 - \lambda_1)(\lambda_8 - \lambda_2)(\lambda_8 - \lambda_3)\dots(\lambda_8 - \lambda_7)} \tag{46}$$

To further facilitate later computations, the following substitutions are made:

$$\begin{array}{ll} a + u = c & rs - pq = t \\ a - u = d & ps + qr = w \\ b + v = g & dh - dg = x \\ b - v = h & d^2 + gh = y \\ dh + dg = p & gc - ch = z \\ cg + ch = q & c^2 + gh = n \\ d^2 - hg = r & yn - xz = m \end{array} \tag{47}$$

$c^2 - hg = s$

$8ab^2t + 8a^2bw = \rho$

$8u^2vk + 8uv^2m = \xi$

$xn + yz = k$

$8a^2bt - 8ab^2w = \sigma$

$8u^2vm - 8uv^2k = \gamma$

Then $R_1 + R_3 + R_6 + R_8$ can be written as follows:

$$\frac{\omega_2^4+\omega_2^4\eta^2-2\omega_2^2\lambda_1^2+\lambda_1^4}{-\rho+i\sigma} + \frac{\omega_2^4+\omega_2^4\eta^2-2\omega_2^2\lambda_3^2+\lambda_3^4}{-\xi+i\gamma}$$
$$+\frac{\omega_2^4+\omega_2^4\eta^2-2\omega_2^2\lambda_6^2+\lambda_6^4}{\rho+i\sigma} + \frac{\omega_2^4+\omega_2^4\eta^2-2\omega_2^2\lambda_8^2+\lambda_8^4}{\xi+i\gamma} \tag{48}$$

The sum of these four terms is a fraction whose denominator is:

$$(-\rho+i\sigma)(\rho+i\sigma)(-\xi+i\gamma)(\xi+i\gamma) = (\rho^2+\sigma^2)(\xi^2+\gamma^2) \tag{49}$$

The numerator is very lengthy. Since the denominator is a real quantity, the numerator must be a purely imaginary quantity, otherwise $E[X^2(t)]$ computed from Equation (8) would be complex. If multiplication of all the terms in the numerator is carried out, it is seen that the real terms do cancel. The numerator so obtained is given as follows:

$$\begin{aligned}\text{NUMERATOR} = \frac{-2iK}{m_1^2}\Big\{ & -4ab\,\omega_2^2\rho\xi^2 + 4a^3b\,\rho\xi^2 \\ & - 4ab^3\rho\xi^2 - 4ab\omega_2^2\rho\gamma^2 + 4a^3b\rho\gamma^2 - 4ab^3\rho\gamma^2 \\ & + \omega_2^4\sigma\xi^2 + \omega_2^2\eta^2\sigma\xi^2 - 2\omega_2^2 a\sigma\xi^2 + 2\omega_2^2b^2\sigma\xi^2 \\ & + a^4\sigma\xi^2 - 6a^2b^2\sigma\xi^2 + b^4\sigma\xi^2 + \omega_2^4\sigma\gamma^2 + \omega_2^2\eta^2\sigma\gamma^2 \\ & - 2\omega_2^2a^2\sigma\gamma^2 + 2\omega_2^2 b^2\sigma\gamma^2 + a^4\sigma\gamma^2 - 6a^2b^2\sigma\gamma^2\end{aligned} \tag{50}$$

$$+ b^4 \sigma \gamma^2 - 4uv\,\omega_2^2 \xi \rho^2 + 4u^3 v\, \xi \rho^2 - 4uv^3 \xi \rho^2$$

$$- 4uv\,\omega_2^2 \xi \sigma^2 + 4u^3 v\, \xi \sigma^2 - 4uv^3 \xi \sigma^2 + \omega_2^4 \gamma \rho^2$$

$$+ \omega_2^2 \eta^2 \gamma \rho^2 - 2\omega_2^2 u^2 \gamma \rho^2 + 2\omega_2^2 v^2 \gamma \rho^2 + u^4 \gamma \rho^2$$

$$- 6u^2 v^2 \gamma \rho^2 + v^4 \gamma \rho^2 + \omega_2^4 \gamma \sigma^2 + \omega_2^2 \eta^2 \gamma \sigma^2$$

$$- 2\omega_2^2 u^2 \gamma \sigma^2 + 2\omega_2^2 v^2 \gamma \sigma^2 + u^4 \gamma \sigma^2 - 6u^2 v^2 \gamma \sigma^2$$

$$\left. + v^4 \gamma \sigma^2 \right\}$$

Therefore:

$$E[X^2(t)] = \frac{2\pi \cdot (\text{NUMERATOR})}{(\rho^2+\sigma^2)(\xi^2+\gamma^2)} \tag{51}$$

Computations involving Equation (51) were carried out on a high speed computer.

The other case that is considered is shown in Figure 2. It is similar to the system shown in Figure 1, but the excitation is the random motion of the foundation. Since the excitation is again a white noise, Equation (8) applies to this case, except that the square of the absolute value of the frequency response function is now given by:

$$|H(\lambda)|^2 = \frac{\lambda^2[\omega_2^2(1+i\eta) - \lambda^2]}{m_1[\lambda^4 - \omega_2^2\lambda^2(1+M) - \omega_1^2\lambda^2 + \omega_1^2\omega_2^2] + i m_1[-\omega_2^2\lambda^2\eta(1+M) + \eta\omega_1^2\omega_2^2]} \cdot \tag{52}$$

$$\frac{\lambda^2[\omega_2^2(1-i\eta) - \lambda^2]}{m_1[\lambda^4 - \omega_2^2\lambda^2(1+M) - \omega_1^2\lambda^2 + \omega_1^2\omega_2^2] - i m_1[-\omega_2^2\lambda^2\eta(1+M) + \eta\omega_1^2\omega_2^2]}$$

This frequency response function refers to the difference between the displacement of mass m_1 and the displacement of the foundation, that is, $X_1 - Y = Z$.

The presence of λ^4 in the numerator of the integrand does not affect the location of the poles of the integrand,

but it does affect the values of the residues. It can be shown that the expression for $E[Z^2(t)]$ can be written as follows:

$$
\begin{aligned}
E[Z^2(t)] = {} & \frac{8\pi K}{m_1^2(\rho^2+\sigma^2)(\xi^2+\gamma^2)} \quad [-4ab\omega_2^2\rho\xi^2 + \\
& 4a^3b\rho\xi^2 - 4ab^3\rho\xi^2 - 4ab\omega_2^2\rho\gamma^2 + 4a^3b\rho\gamma^2 - 4ab^3\rho\gamma^2 \\
& +\omega_2^4\sigma\xi^2 + \omega_2^2\eta^2\sigma\xi^2 - 2\omega_2^2a^2\sigma\xi^2 + 2\omega_2^2b^2\sigma\xi^2 \\
& + a^4\sigma\xi^2 - 6a^2b^2\sigma\xi^2 + b^4\sigma\xi^2 + \omega_2^4\sigma\gamma^2 + \omega_2^2\eta^2\sigma\gamma^2 \\
& - 2\omega_2^2a^2\sigma\gamma^2 + 2\omega_2^2b^2\sigma\gamma^2 + a^4\sigma\gamma^2 - 6a^2b^2\sigma\gamma^2 \\
& + b^4\sigma\gamma^2]\cdot[a^4 - 6a^2b^2 + b^4] + [-4uv\omega_2^2\xi\rho^2 + 4u^3v\xi\rho^2 \\
& - 4uv^3\xi\rho^2 - 4uv\omega_2^2\xi\sigma^2 + 4u^3v\xi\sigma^2 - 4uv^3\xi\sigma^2 \\
& +\omega_2^4\gamma\rho^2 + \omega_2^2\eta^2\gamma\rho^2 - 2\omega_2^2u^2\gamma\rho^2 + 2\omega_2^2v^2\gamma\rho^2 + \\
& u^4\gamma\rho^2 - 6u^2v^2\gamma\rho^2 + v^4\gamma\rho^2 + \omega_2^4\gamma\sigma^2 + \omega_2^2\eta^2\gamma\sigma^2 \\
& - 2\omega_2^2u^2\gamma^2\sigma^2 + 2\omega_2^2v^2\gamma\sigma^2 + u^4\gamma\sigma^2 - 6u^2v^2\gamma\sigma^2 \\
& + v^4\gamma\sigma^2]\cdot[u^4 - 6u^2v^2 + v^4] + [4u^3v - 4uv^3]\cdot \\
& [\xi\rho^2\omega_2^4 + \xi\rho^2\omega_2^2\eta^2 - 2\xi\rho^2\omega_2^2u^2 + 2\xi\rho^2\omega_2^2v^2 \\
& +\xi\rho^2u^4 - 6\xi\rho^2u^2v^2 + \xi\rho^2v^4 + \xi\sigma^2\omega_2^4 + \xi\sigma^2\omega_2^2\eta^2 \\
& - 2\xi\sigma^2\omega_2^2u^2 + 2\xi\sigma^2\omega_2^2v^2 + \xi\sigma^2u^4 - 6\xi\sigma^2u^2v^2 \\
& + \xi\sigma^2v^4 + 4\gamma\rho^2uv\omega_2^2 - 4\gamma\rho^2u^3v + 4\gamma\rho^2uv^3 + \\
& 4\gamma\sigma^2uv\omega_2^2 - 4\gamma\sigma^2u^3v + 4\gamma\sigma^2uv^3] + [4a^3b - 4ab^3]\cdot \\
& [\rho\xi^2\omega_2^4 + \rho\xi^2\omega_2^2\eta^2 - 2\rho\xi^2\omega_2^2a^2 + 2\rho\xi_2^2\, b^2 +
\end{aligned}
\tag{53}
$$

$$\rho\xi^2 a^4 - 6\rho\xi^2 a^2 b^2 + \rho\xi^2 b^4 + \rho\gamma^2\omega_2^4 + \rho\gamma^2\omega_2^2\eta^2 -$$
$$2\rho\gamma^2\omega_2^2 a^2 + 2\rho\gamma^2\omega_2^2 b^2 + \rho\gamma^2 a^4 - 6\rho\gamma^2 a^2 b^2 +$$
$$\rho\gamma^2 b^4 + 4\sigma\xi^2\omega_2^2 ab - 4\sigma\xi^2 a^3 b + 4\sigma\xi^2 ab^3$$
$$+ 4\sigma\gamma^2\omega_2^2 ab - 4\sigma\gamma^2 a^3 b + 4\sigma\gamma^2 ab^3]$$

Computations involving Equation (53) were also carried out on a high speed computer.

3. RESULTS

The results of this paper are presented in both tabular and graphical forms. For all cases considered, m_1 and K are set equal to unity.

Table 1 lists values of the mass ratio M of 0.02, 0.05, 0.1, and 0.2; typical values of the loss factor η of 0.2, 0.5, and 1.0; and values of the natural frequency ω_1 of the primary system of 10, 20, 30, 40, and 50 radians per second. For each combination of the above parameters, there are listed two values of the natural frequency ω_2 of the damper unit. The first column of ω_2 represents the optimum ω_2 for minimum mean square response when the system is excited by a white noise at m_1. The second column of ω_2 represents the optimum ω_2 for minimum mean square response when the foundation is moved in a random fashion.

Table 2 lists values of the mass ratio M of 0.05, 0.1, and 0.2; values of the loss factor η of 0.2 and 0.5; and values of the natural frequency ω_1 of the primary system of 10, 20, 30, 40, and 50 radians per second. The first column of ω_2 represents the optimum ω_2 for the equal peak criterion when the excitation is applied at m_1. The second column of ω_2 represents the optimum ω_2 for the equal peak criterion when the excitation is the motion of the foundation. Data for the equal peak criterion was obtained for only a limited number of combinations of the parameters. For the combinations of parameters listed in Table 1 that are not listed in Table 2, the absolute value of the frequency

response function exhibited essentially only one peak. The cases where all four sets of optimum ω_2 were obtained are plotted in Figures 4, 5, 6, and 7.

To further contrast the different results obtained by use of two different tuning criteria, in Figures 8, 9 and 10 are plotted the square of the absolute value of the frequency response function versus ω for a mass ratio M = 0.05, a loss factor η = 0.2, and a natural frequency of the primary system ω_1 = 40 radians per second. Figure 8 represents the case of ω_2 = 38.4 radians per second, which is the value of ω_2 that satisfies the equal peak criterion. Figure 9 represents the case of ω_2 = 38.9 radians per second, which is the value of ω_2 that satisfies the minimum mean square response criterion when the system is excited by a white noise at m_1. This value of ω_2 causes the first peak to be of greater amplitude than the second peak. Figure 10 represents the case of ω_2 = 38.0 radians per second. This value of ω_2 is less than both the frequency values corresponding to the two tuning criteria stated above, and it causes the second peak to be of greater amplitude than the first peak.

Table 3 lists values of loss factor η of 0.07, 0.10, 0.20, 0.50, and 1.0 in combination with a single mass ratio M = 0.02 and a single natural frequency of the primary system ω_1 = 50 radians per second. For each η value, nine values of ω_2 are listed together with their respective values of mean square response for the case of white noise excitation

at m_1. The units on $E[X^2(t)]$ would depend on the units used for the masses in the system. If m_1 and m_2 are specified by the units pound-second^2 per inch, then $E[X^2(t)]$ would be expressed in terms of inch^2. The data from Table 3 is plotted in Figure 11.

Table 4 contains the same combinations of parameters as Table 3, except that Table 4 refers to the case of random motion of the foundation. The data of Table 4 is plotted in Figure 12.

Table 5 lists values of the mass ratio M of 0.02, 0.05, 0.10, and 0.20 in combination with values of loss factor η of 0.04, 0.07, 0.10, 0.20, 0.50, and 1.00, and a single ω_1 = 10 radians per second. For each mass ratio and the given ω_1, there exists an optimum loss factor giving rise to a minimum value of mean square response for each case of white noise excitation. The data for the case where white noise excitation is applied at m_1 is plotted in Figure 13, and the data for the case where white noise excitation is applied at the foundation is plotted in Figure 14.

Table 6 is identical to that of Table 5 except that the natural frequency ω_1 of the primary system is 50 radians per second. The data from Table 6 is plotted in Figures 15 and 16.

4. CONCLUSIONS

Figures 5, 6, and 7 show that when the excitation is applied at m_1, the two criteria for optimization give rise to optimum values of ω_2 that are not identical, but very close to each other. When the excitation is the motion of the foundation, the two criteria also give rise to optimum values of ω_2 that are very close to each other.

It is also seen that the plots of ω_1 versus optimum ω_2 are essentially linear. By comparing Figures 6 and 7, it is found that for a given mass ratio, the slope of the plots decreases as loss factor decreases. This implies that the greater the loss factor the softer the damper unit must be to satisfy either optimum criterion.

Figures 8, 9, and 10 show that the second peak is higher than the first peak for a value of ω_2 that is smaller than the one giving rise to equal peaks. The reverse is true if ω_2 is larger than that which gives rise to equal peaks. If ω_2 is considerably smaller than or considerably larger than the optimum ω_2, essentially only one peak will appear in the absolute value of the frequency response function.

As mentioned previously in this section, Figures 4, 5, 6, and 7 show that the two criteria for optimization are not exactly identical. Figures 11 and 12, however, show that the plots representing mean square response versus ω_2 are quite flat in the region of their respective minima. It can be seen that for a given value of mass ratio and a given value of ω_1, the greater the loss factor, the more slowly the mean square

response varies in the region of its minimum. For the case of mass ratio M = 0.2, loss factor η = 0.2, and ω_1 = 50 radians per second, the equal peak condition is accompanied by a mean square response which is only two per cent larger than the minimum mean square response.

Figures 11 and 12 show that as the value of ω_2 deviates from the optimum value of ω_2, the mean square response increases. For the limiting cases, that is, $\omega_2 = 0$ and $\omega_2 = \infty$, the mean square response is unbounded. The case of $\omega_2 = 0$ corresponds to the case where the damper unit is not attached to the primary system. Since the primary system is of a single degree-of-freedom in which no damping is present, the frequency response function becomes unbounded at $\omega = \omega_1$. For this case the area under the squared frequency response function curve is also infinite. For the case of $\omega_2 = \infty$, the damper spring is infinitely stiff so that the viscoelastic link becomes a rigid link, and again the system degenerates into a single degree-of-freedom system. The mass of this system is made up of m_1 plus m_2 joined together by the rigid link. Similar to the case of $\omega_2 = 0$, the squared frequency response function becomes unbounded at some frequency and results in an infinte area under the curve.

Referring to Figures 13, 14, 15, and 16 it can be seen that for each combination of mass ratio and ω_1, there exists a value of the loss factor that will give rise to a minimum mean square response. For a given value of ω_1, the optimum loss factor decreases as the mass ratio decreases. The

optimum loss factor for a given mass ratio does not appear to be a function of ω_1 as can be seen by comparing Figure 12 with Figure 14 and Figure 13 with Figure 15. Data for the case of $\omega_1 = 30$ radians per second also verifies this. However, the value of the minimum mean square response corresponding to the optimum loss factor does appear to be a function of ω_1. This can be seen by again comparing Figures 12 and 14 and Figures 13 and 15.

In short, it can be said that for a combination of loss factor, mass ratio, and ω_1, such that the equal peak condition can be satisfied, the optimum ω_2 corresponding to equal peaks is almost equal to that which satisfies the minimum mean square response criterion. However, it should be noted that there are combinations of the above parameters that allow the system to be optimized according to minimum mean square response but do not allow the system to be optimized according to the equal peak criterion.

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TABLE 1

Optimum Values of ω_2 for Minimum Mean Square Response

M	η	ω_1	ω_2 for white noise at m_1	ω_2 for white noise at foundation
0.02	0.2	10	9.80	9.95
		20	19.6	19.9
		30	29.4	29.8
		40	39.3	39.7
		50	49.2	49.5
	0.5	10	9.36	9.49
		20	18.6	19.0
		30	28.3	28.6
		40	37.7	38.2
		50	47.1	47.8
	1.0	10	8.37	8.39
		20	16.5	16.6
		30	25.0	25.2
		40	33.8	33.9
		50	43.5	44.2
0.05	0.2	10	9.74	10.0
		20	19.5	19.9
		30	29.2	29.9
		40	38.9	39.75
		50	47.8	49.6
	0.5	10	9.22	9.60
		20	18.5	19.1
		30	27.9	28.7
		40	37.1	38.2
		50	46.8	47.9
	1.0	10	8.22	8.40
		20	16.3	16.7
		30	24.8	25.3
		40	33.1	33.6
		50	41.4	42.5

TABLE 1 (continued)

Optimum Values of ω_2 for Minimum Mean Square Response

M	η	ω_1	ω_2 for white noise at m_1	ω_2 for white noise at foundation
0.1	0.2	10	9.65	10.05
		20	19.2	20.2
		30	28.7	30.1
		40	38.4	40.1
		50	47.1	50.1
	0.5	10	9.05	9.60
		20	18.2	19.2
		30	27.4	28.8
		40	36.2	38.4
		50	45.6	48.0
	1.0	10	8.12	8.52
		20	16.3	16.9
		30	24.2	25.7
		40	32.4	33.9
		50	41.0	42.7
0.2	0.2	10	9.28	10.2
		20	18.6	20.5
		30	27.9	30.8
		40	37.1	41.2
		50	46.5	51.2
	0.5	10	8.90	9.70
		20	17.7	19.6
		30	26.6	29.5
		40	35.4	39.3
		50	43.7	49.0
	1.0	10	8.00	8.66
		20	15.8	17.3
		30	23.9	25.8
		40	31.1	34.2
		50	38.8	43.4

TABLE 2

Optimum Values of ω_2 for Equal Peak Criterion

M	η	ω_1	ω_2 for sinusoidal input at m_1	ω_2 for sinusoidal input at foundation
0.05	0.2	10	9.60	9.90
		20	19.2	19.8
		30	28.7	29.6
		40	38.4	39.5
		50	48.0	49.4
0.1	0.2	10	9.40	10.0
		20	18.8	19.9
		30	28.1	29.7
		40	37.6	39.6
		50	46.9	49.5
0.2	0.2	10	9.10	9.90
		20	17.9	19.9
		30	26.7	29.8
		40	35.9	39.7
		50	44.9	49.6
	0.5	10	8.50	9.50
		20	16.9	18.7
		30	25.4	27.9
		40	33.9	37.3
		50	42.6	46.9

TABLE 3

Mean Square Response for the Case of White Noise Excitation at m_1

M	η	ω_1	ω_2	$E[X^2(t)] \times 10^3$
0.02	0.07	50	37.4	4.97
			40.0	3.92
			42.45	3.00
			44.75	1.01
			46.9	0.68
			49.0	0.46
			51.0	0.51
			52.9	0.71
			54.8	1.47
	0.10	50	37.4	3.88
			40.0	2.85
			42.45	1.99
			44.75	0.84
			46.9	0.53
			49.0	0.38
			51.0	0.42
			52.9	0.60
			54.8	0.97
	0.20	50	37.4	1.58
			40.0	1.43
			42.45	0.98
			44.75	0.61
			46.9	0.43
			49.0	0.37
			51.0	0.41
			52.9	0.51
			54.8	0.67
	0.50	50	37.4	1.06
			40.0	0.93
			42.45	0.76
			44.75	0.67
			46.9	0.65
			49.0	0.67
			51.0	0.72
			52.9	0.81
			54.8	0.93
	1.00	50	37.4	1.15
			40.0	1.11
			42.45	1.07
			44.75	1.08
			46.9	1.11
			49.0	1.16
			51.0	1.23
			52.9	1.30
			54.8	1.38

TABLE 4

Mean Square Response for the Case of White Noise Excitation at the Foundation

M	η	ω_1	ω_2	$E[Z^2(t)] \times 10^{-4}$
0.02	0.07	50	37.4	
			40.0	
			42.45	2.03
			44.75	0.71
			46.9	0.48
			49.0	0.30
			51.0	0.298
			52.9	0.394
			54.8	0.797
	0.10	50	37.4	
			40.0	
			42.45	1.34
			44.75	0.58
			46.9	0.365
			49.0	0.246
			51.0	0.245
			52.9	0.334
			54.8	0.533
	0.20	50	37.4	1.03
			40.0	0.95
			42.45	0.65
			44.75	0.40
			46.9	0.28
			49.0	0.23
			51.0	0.24
			52.9	0.29
			54.8	0.38
	0.50	50	37.4	0.76
			40.0	0.59
			42.45	0.48
			44.75	0.42
			46.9	0.40
			49.0	0.41
			51.0	0.43
			52.9	0.48
			54.8	0.55
	1.00	50	37.4	0.76
			40.0	0.68
			42.45	0.654
			44.75	0.652
			46.9	0.67
			49.0	0.70
			51.0	0.73
			52.9	0.78
			54.8	0.82

TABLE 5

Minimum Mean Square Response for Both Cases of White Noise Excitation

M	η	ω_1	$E[X^2(t)] \times 10^1$ excitation at m_1	$E[Z^2(t)] \times 10^{-3}$ excitation at foundation
0.02	0.04	10	0.916	0.921
	0.07		0.56	0.58
	0.10		0.47	0.488
	0.20		0.473	0.485
	0.50		0.816	0.818
	1.00		1.35	1.34
0.05	0.04	10	0.816	0.822
	0.07		0.501	0.514
	0.10		0.385	0.396
	0.20		0.283	0.298
	0.50		0.369	0.372
	1.00		0.568	0.562
0.10	0.04	10	0.785	0.856
	0.07		0.467	0.499
	0.10		0.343	0.369
	0.20		0.220	0.236
	0.50		0.219	0.223
	1.00		0.309	0.301
0.20	0.20	10	0.186	0.207
	0.50		0.144	0.148
	1.00		0.178	0.169

TABLE 6

Minimum Mean Square Response for Both Cases of White Noise Excitation

M	η	ω_1	$E[X^2(t)] \times 10^3$ excitation at m_1	$E[Z^2(t)] \times 10^{-4}$ excitation at foundation
0.02	0.04	50	0.63	0.39
	0.07		0.44	0.27
	0.10		0.38	0.23
	0.20		0.37	0.23
	0.50		0.65	0.40
	1.00		1.07	0.65
0.05	0.04	50	0.61	0.41
	0.07		0.38	0.24
	0.10		0.29	0.186
	0.20		0.22	0.137
	0.50		0.29	0.175
	1.00		0.45	0.269
0.10	0.04	50	0.60	0.39
	0.07		0.37	0.23
	0.10		0.27	0.172
	0.20		0.172	0.105
	0.50		0.174	0.09
	1.00		0.244	0.138
0.20	0.20	50	0.146	0.087
	0.50		0.112	0.06
	1.00		0.14	0.072

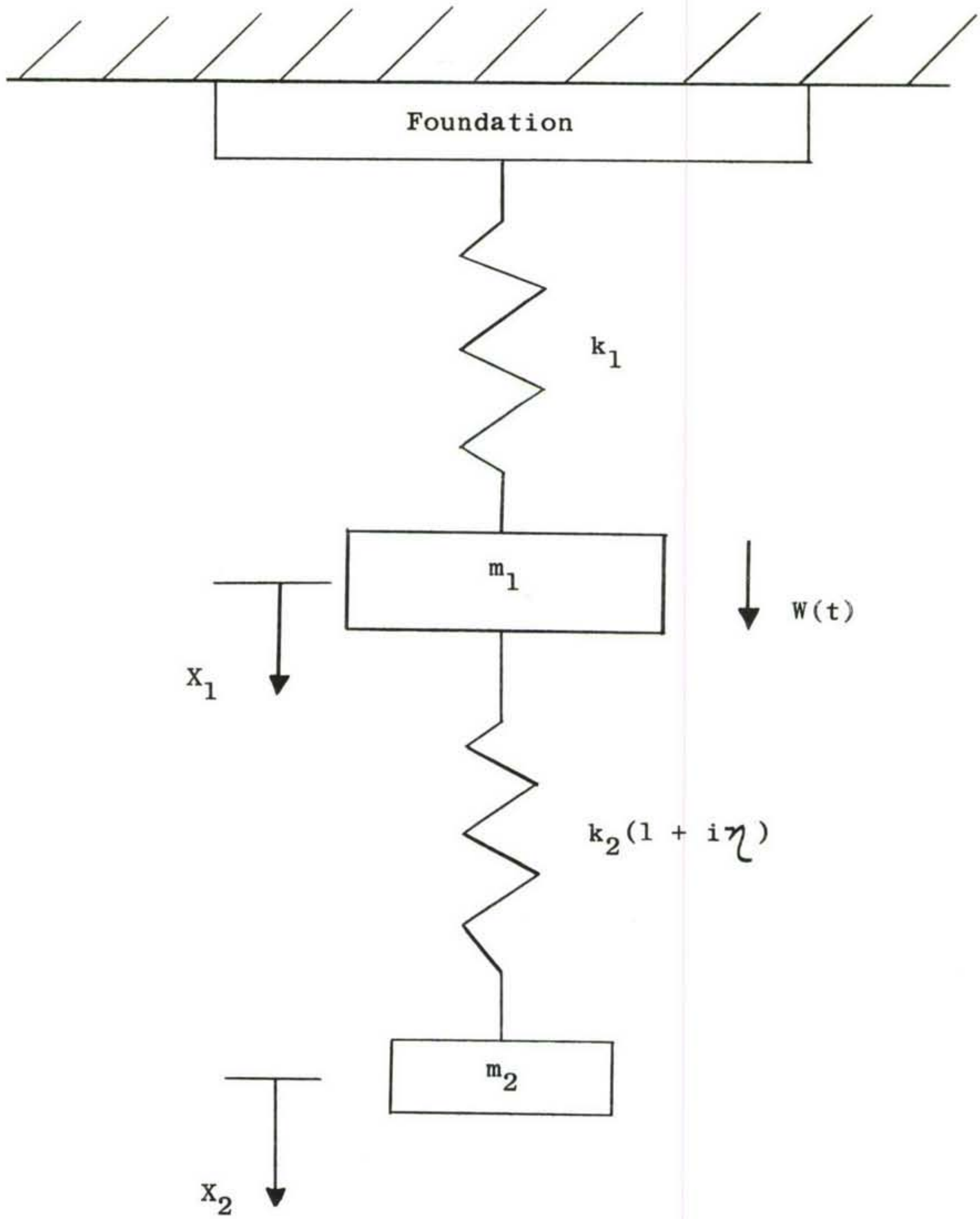

Figure 1. Random excitation at m_1

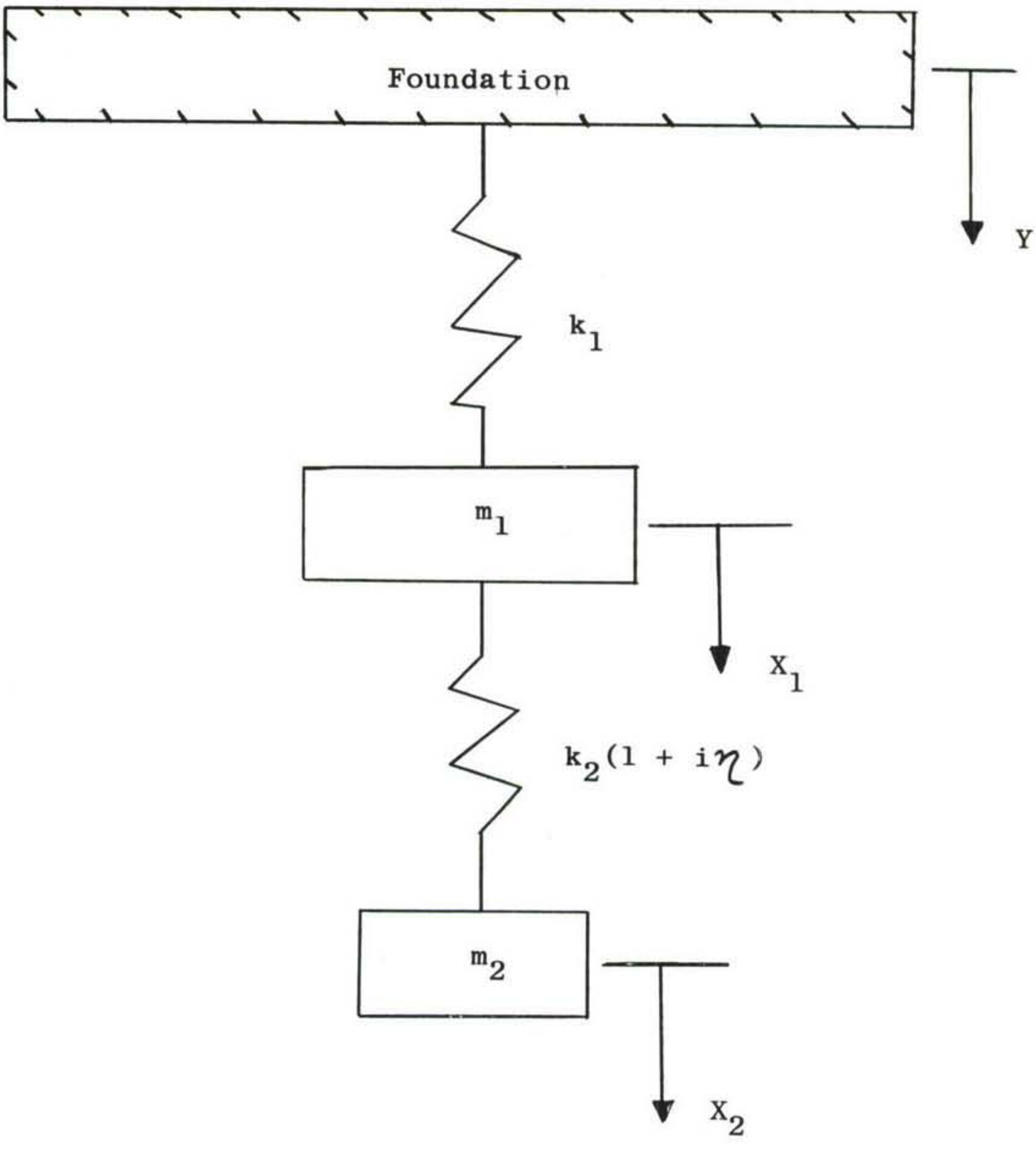

Figure 2. Random motion of foundation

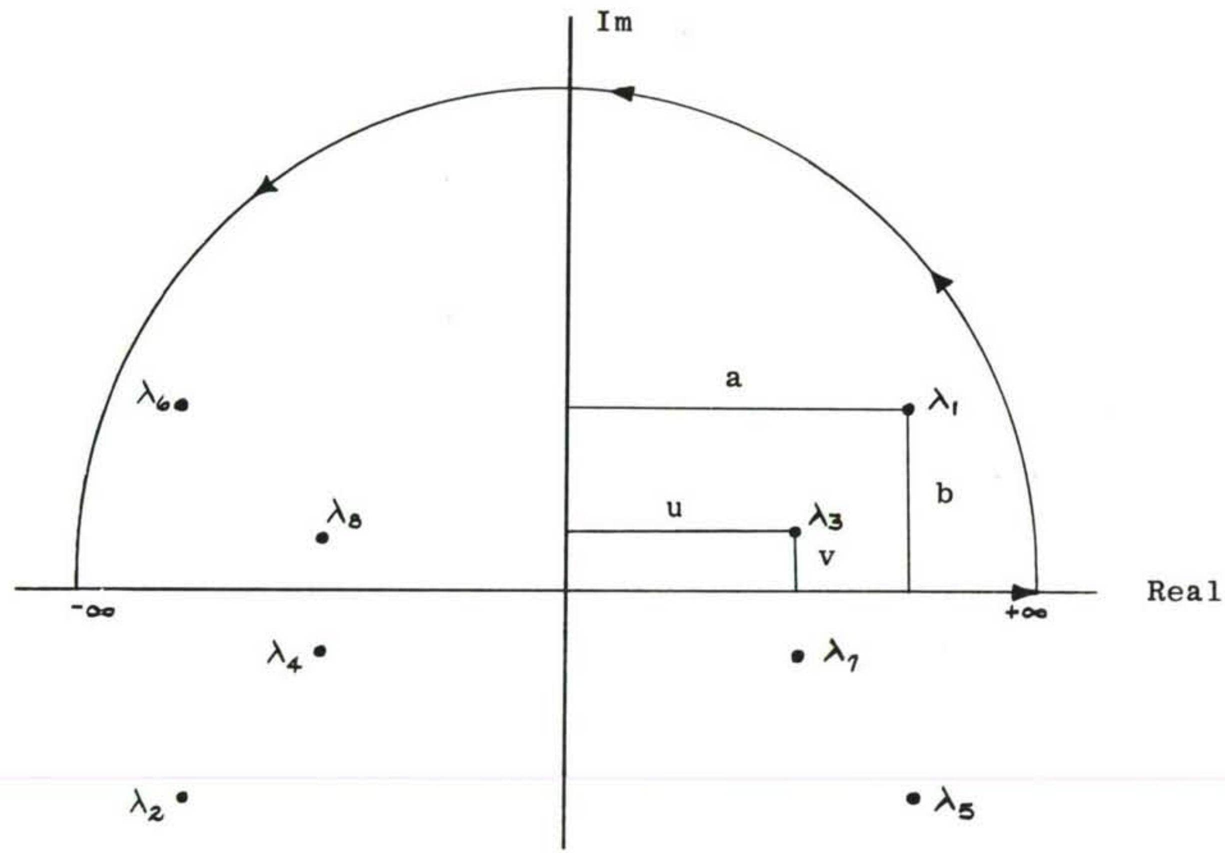

Figure 3. Location of poles and path used in contour integration

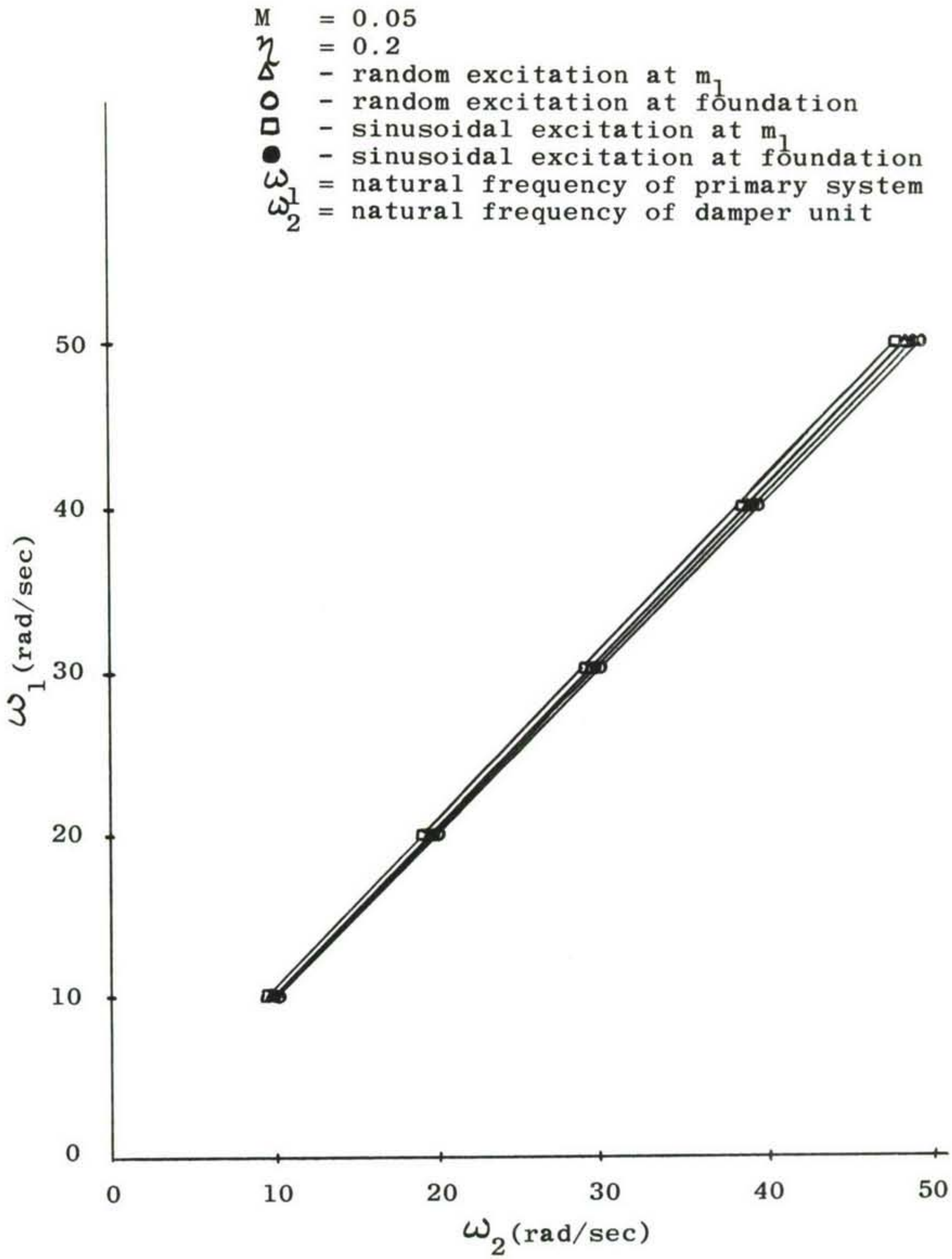

Figure 4. Plot of ω_1 versus optimum ω_2

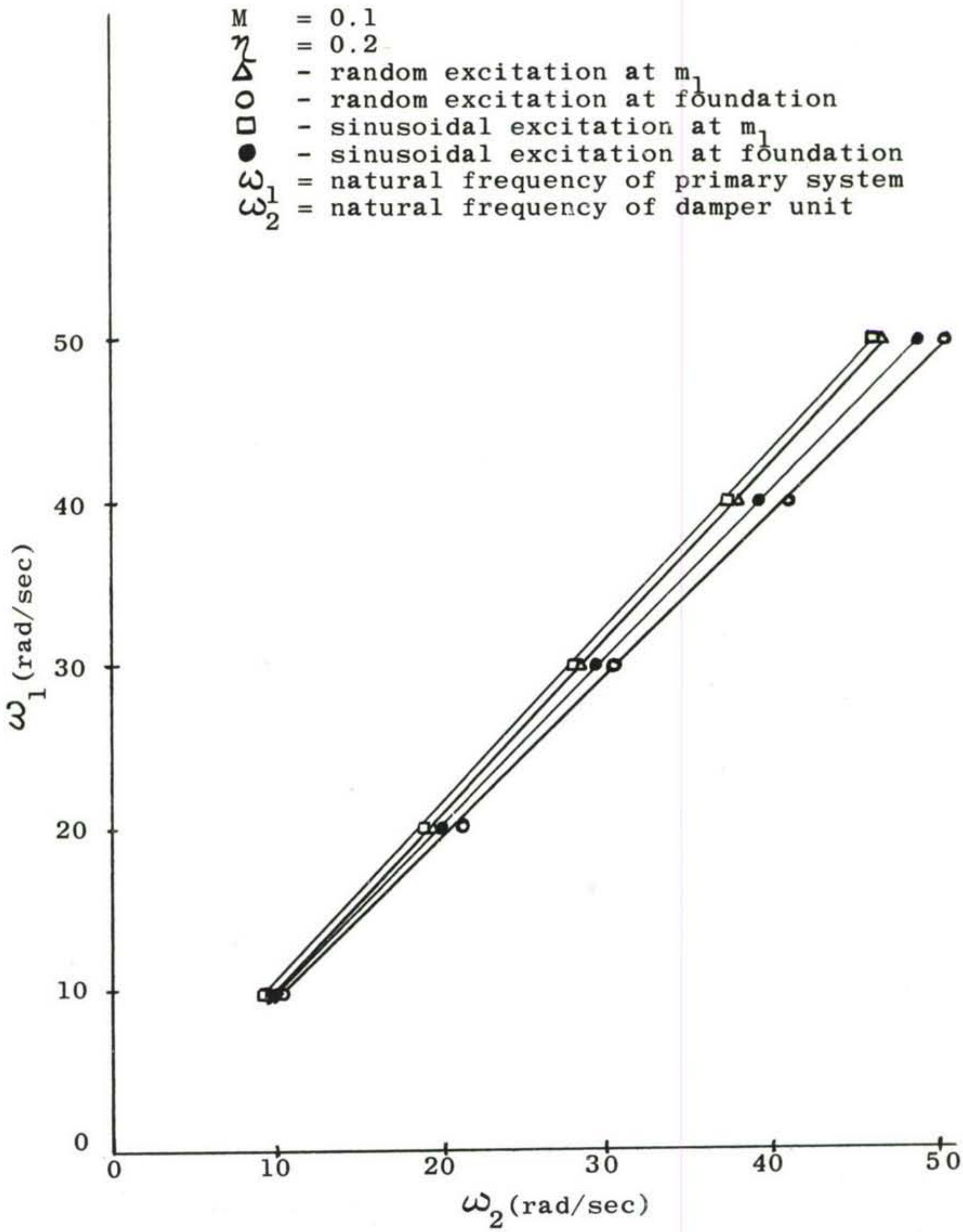

Figure 5. Plot of ω_1 versus optimum ω_2

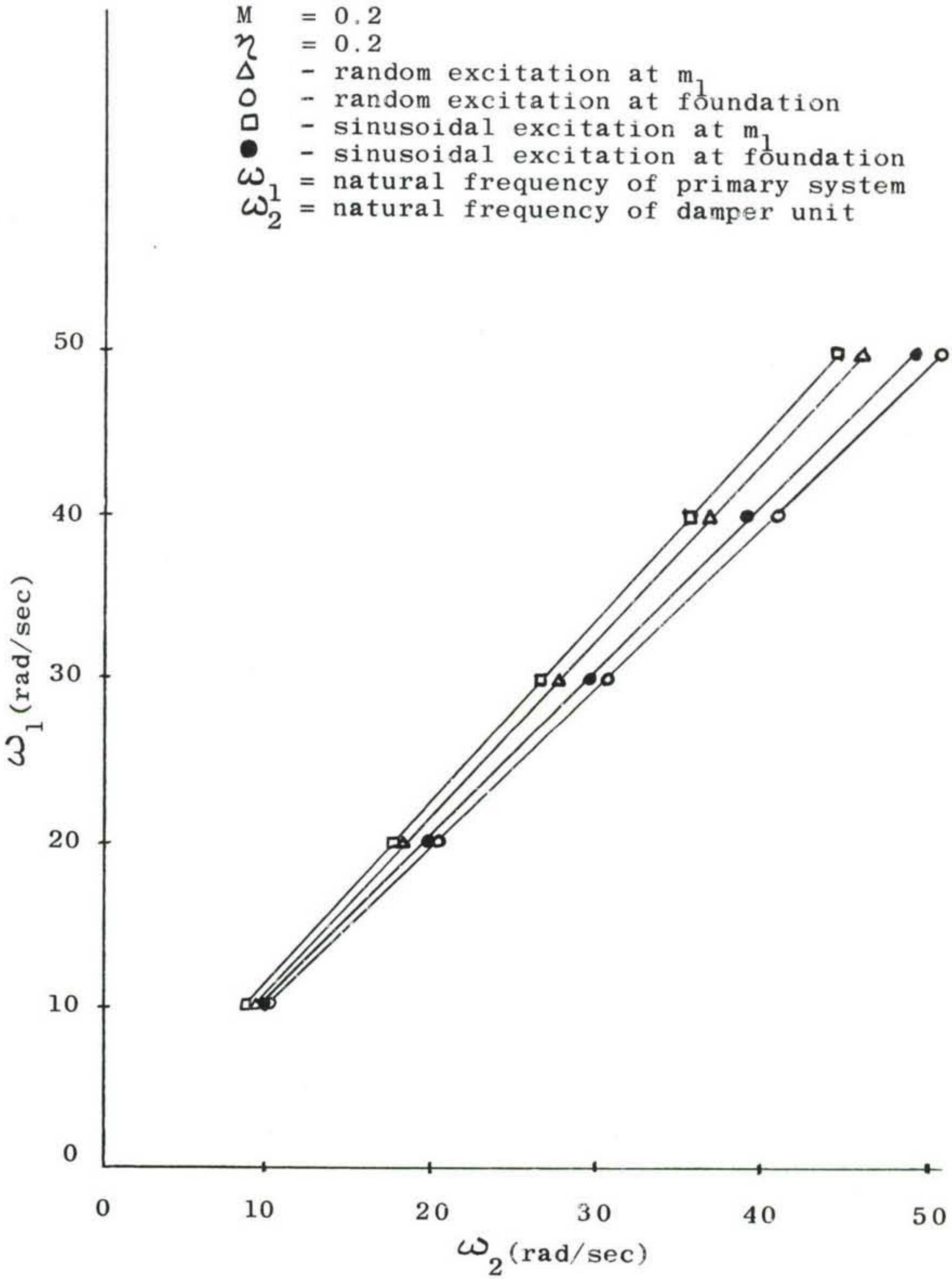

Figure 6. Plot of ω_1 versus optimum ω_2

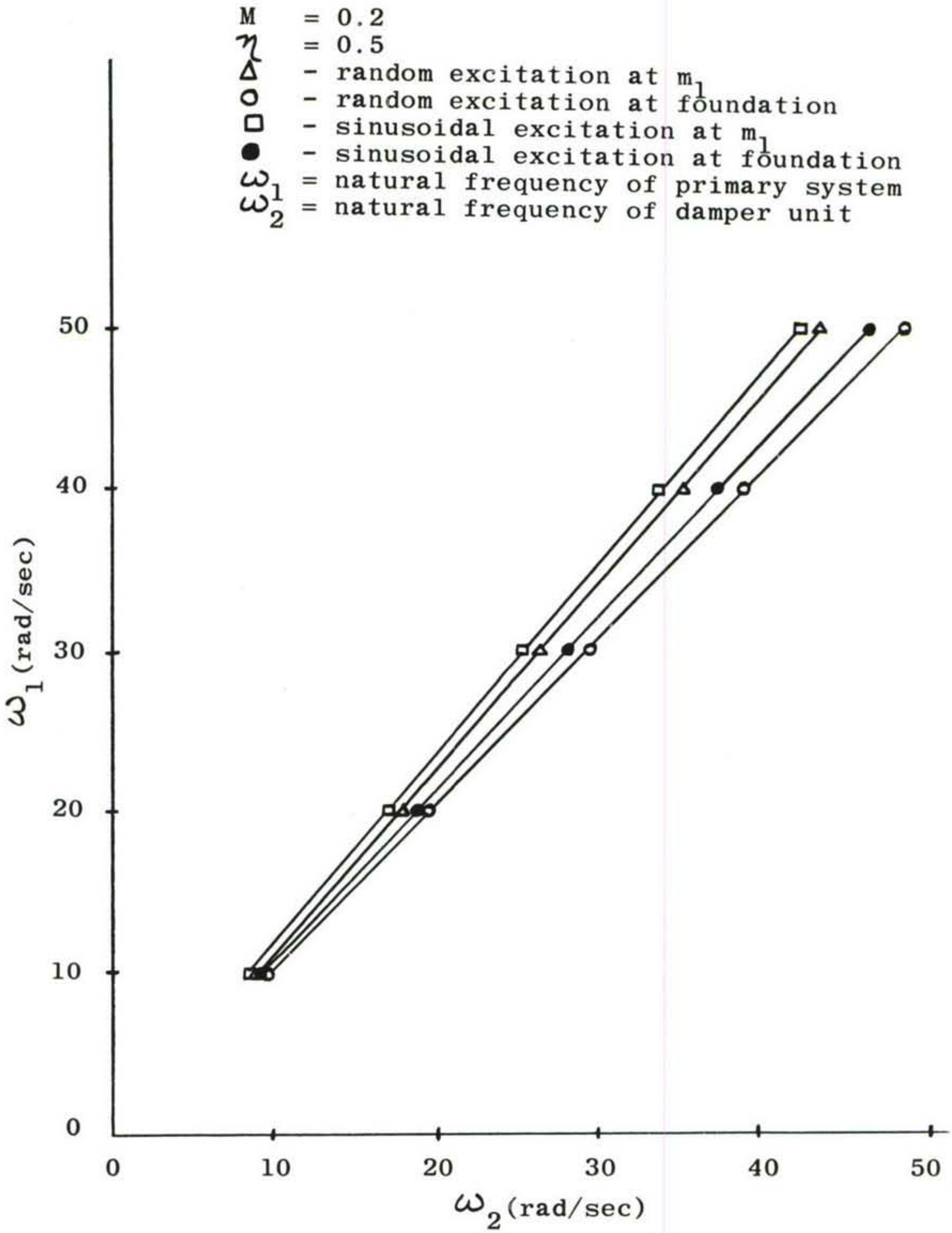

Figure 7. Plot of ω_1 versus optimum ω_2

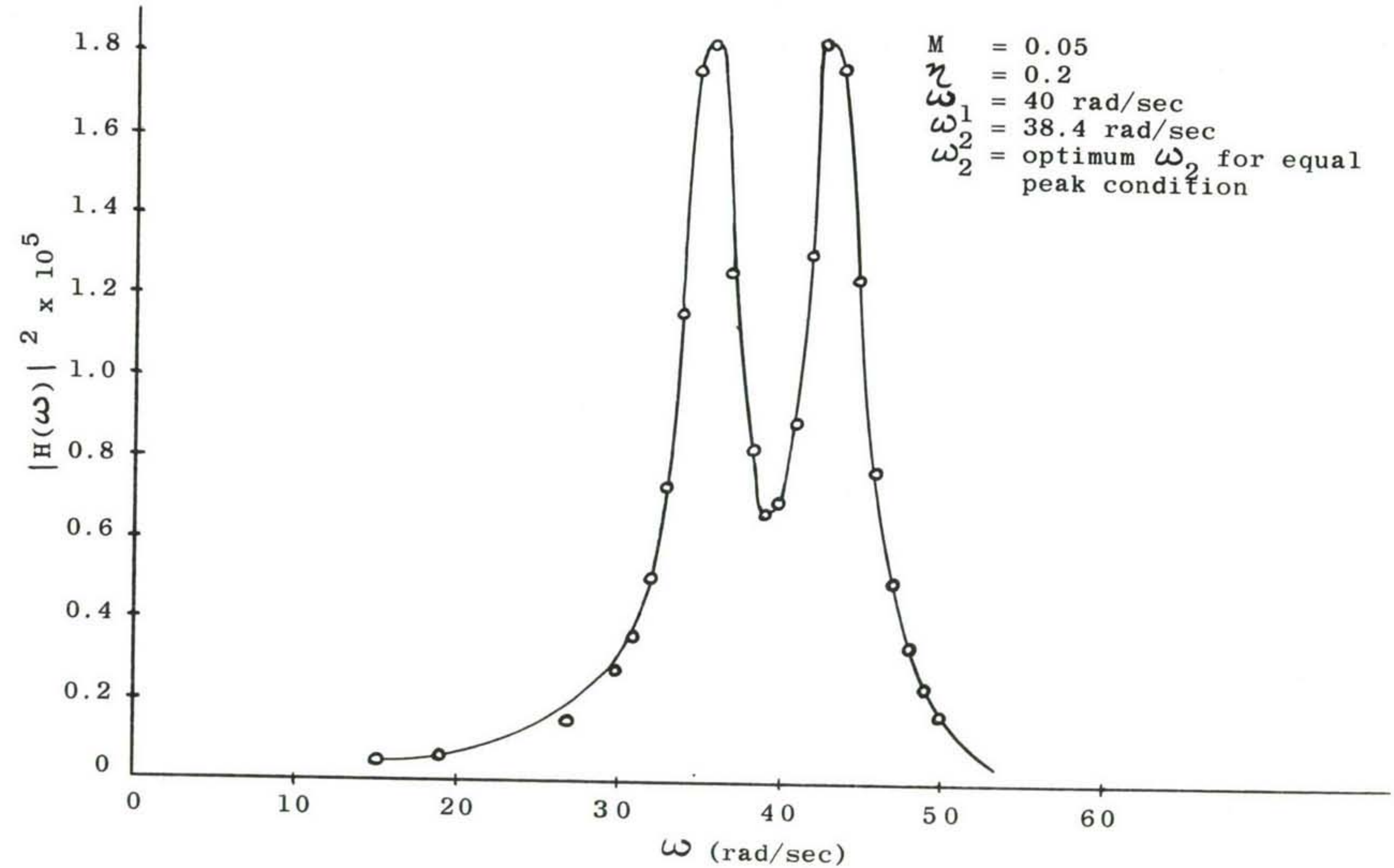

Figure 8. Plot of $|H(\omega)|^2$ versus ω (excitation at m_1)

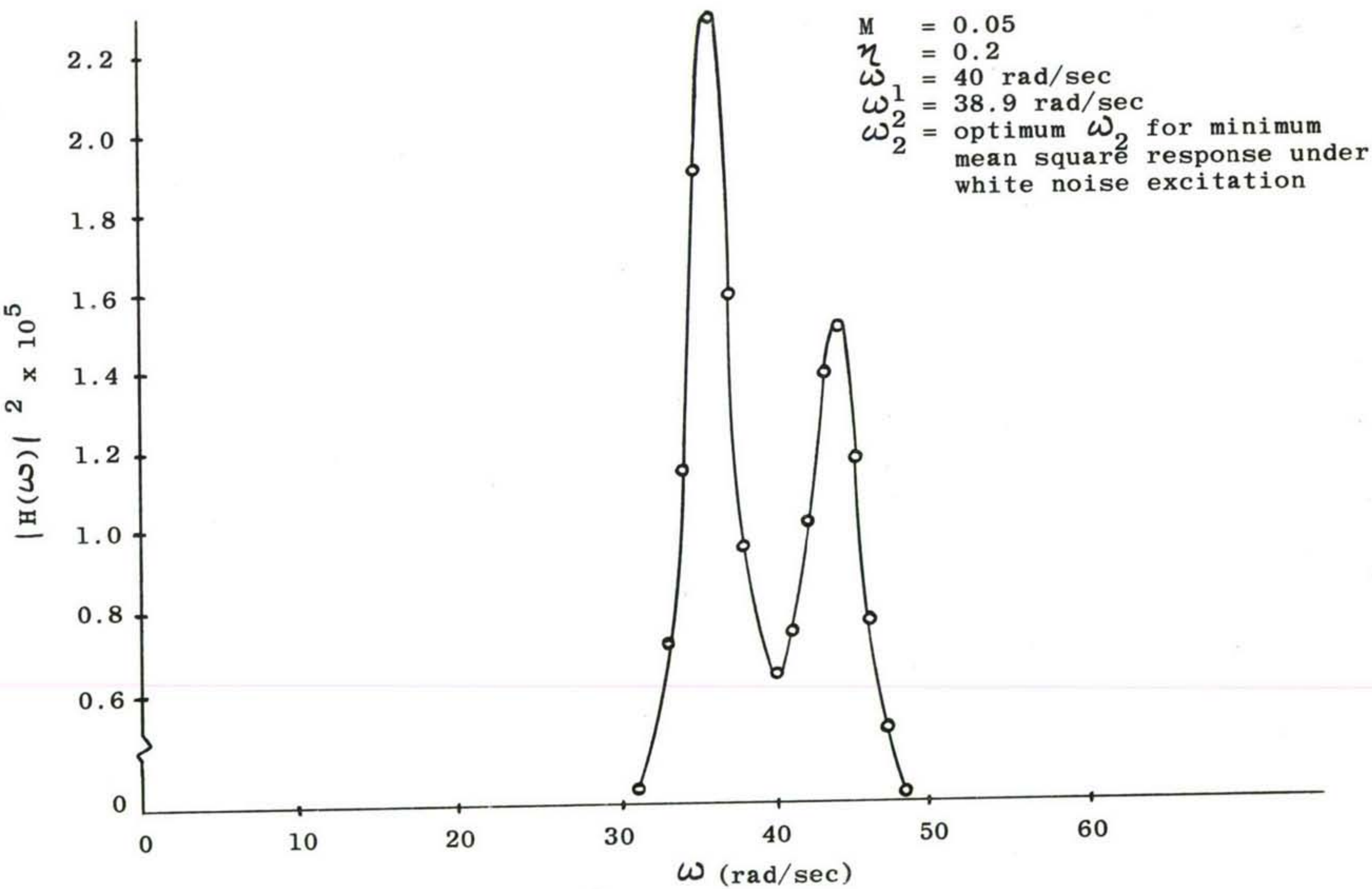

Figure 9. Plot of $|H(\omega)|^2$ versus ω (excitation at m_1)

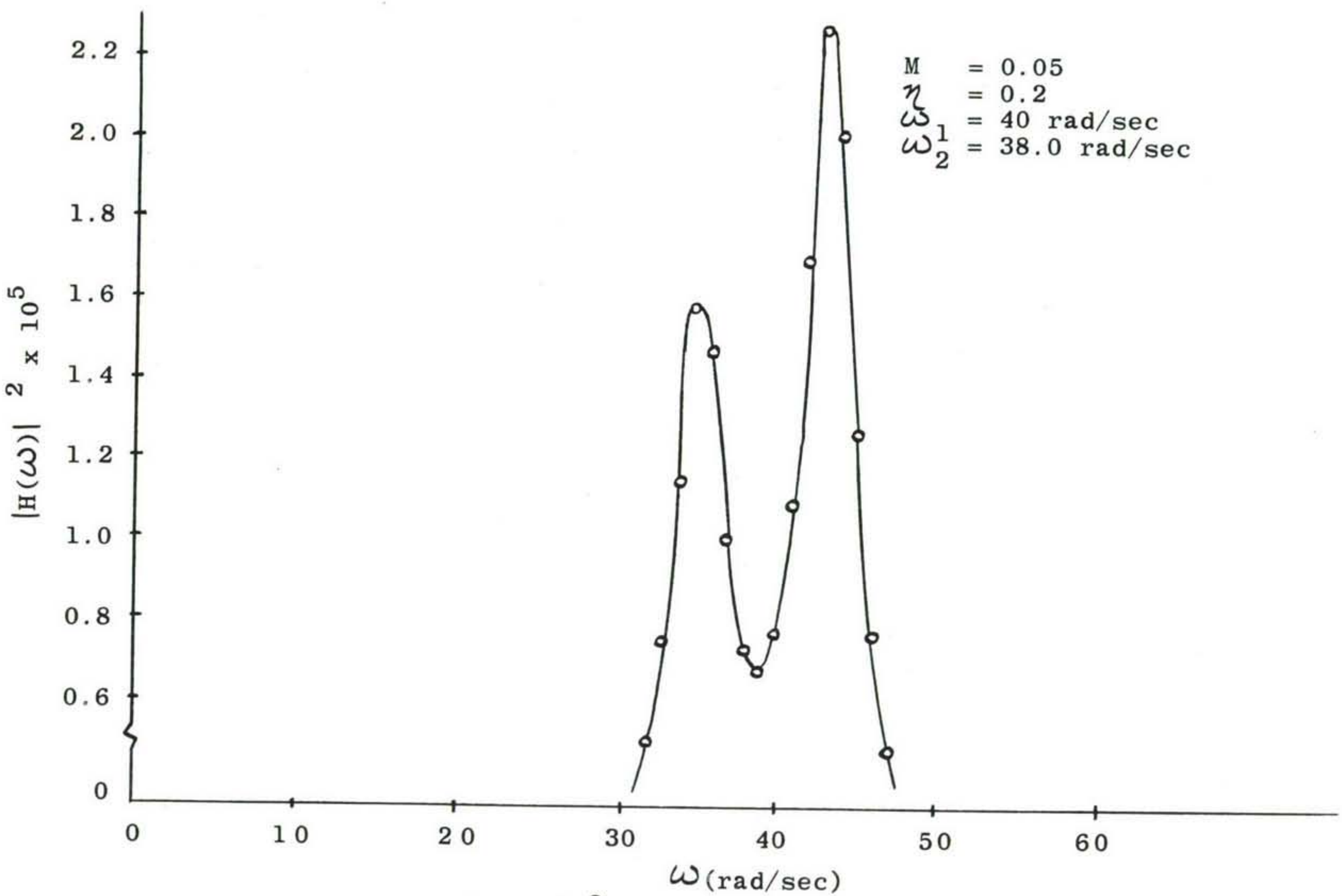

Figure 10. Plot of $|H(\omega)|^2$ versus ω (excitation at m_1)

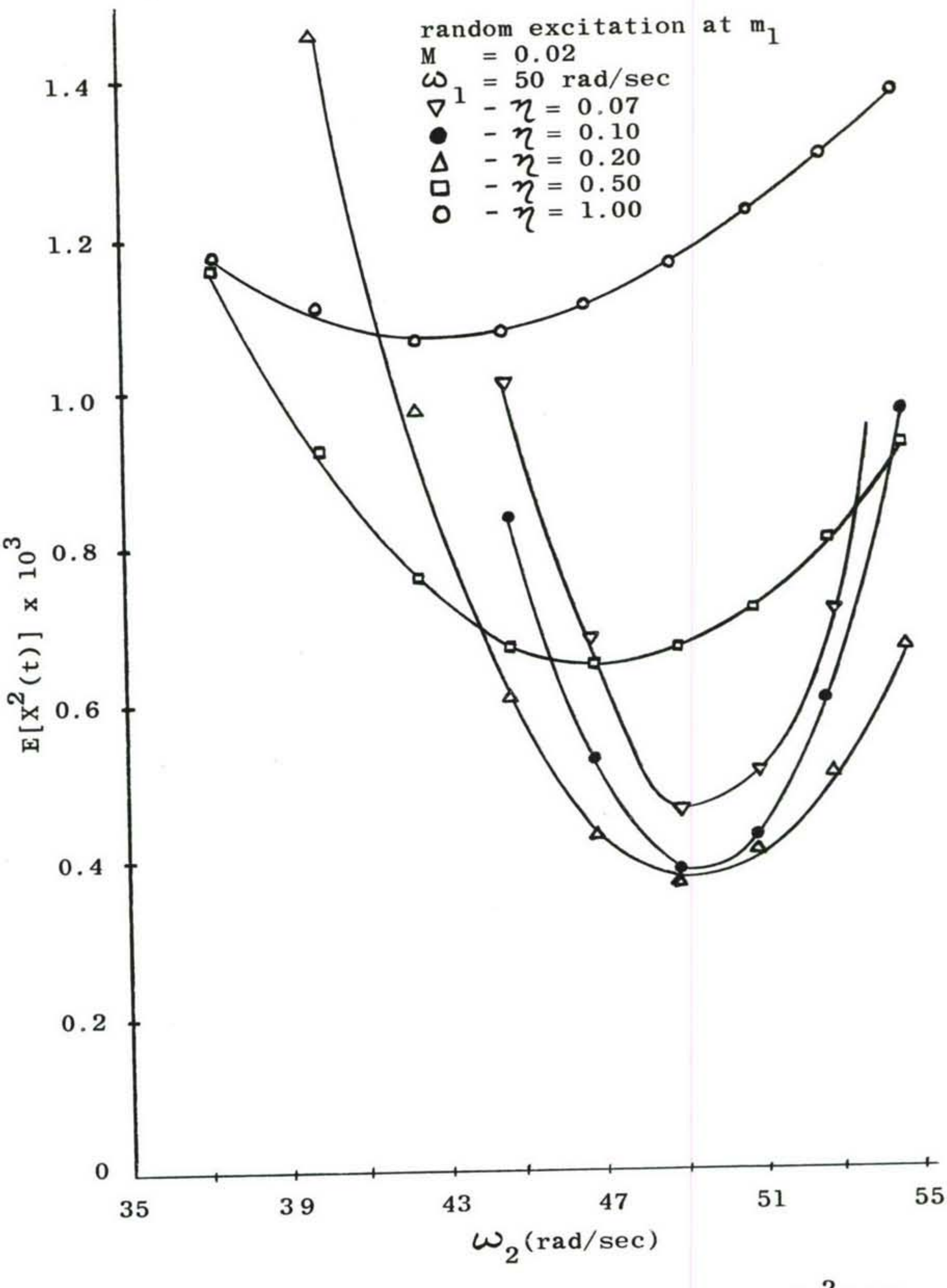

Figure 11. Plot of mean square response $E[X^2(t)]$ versus natural frequency of the damper

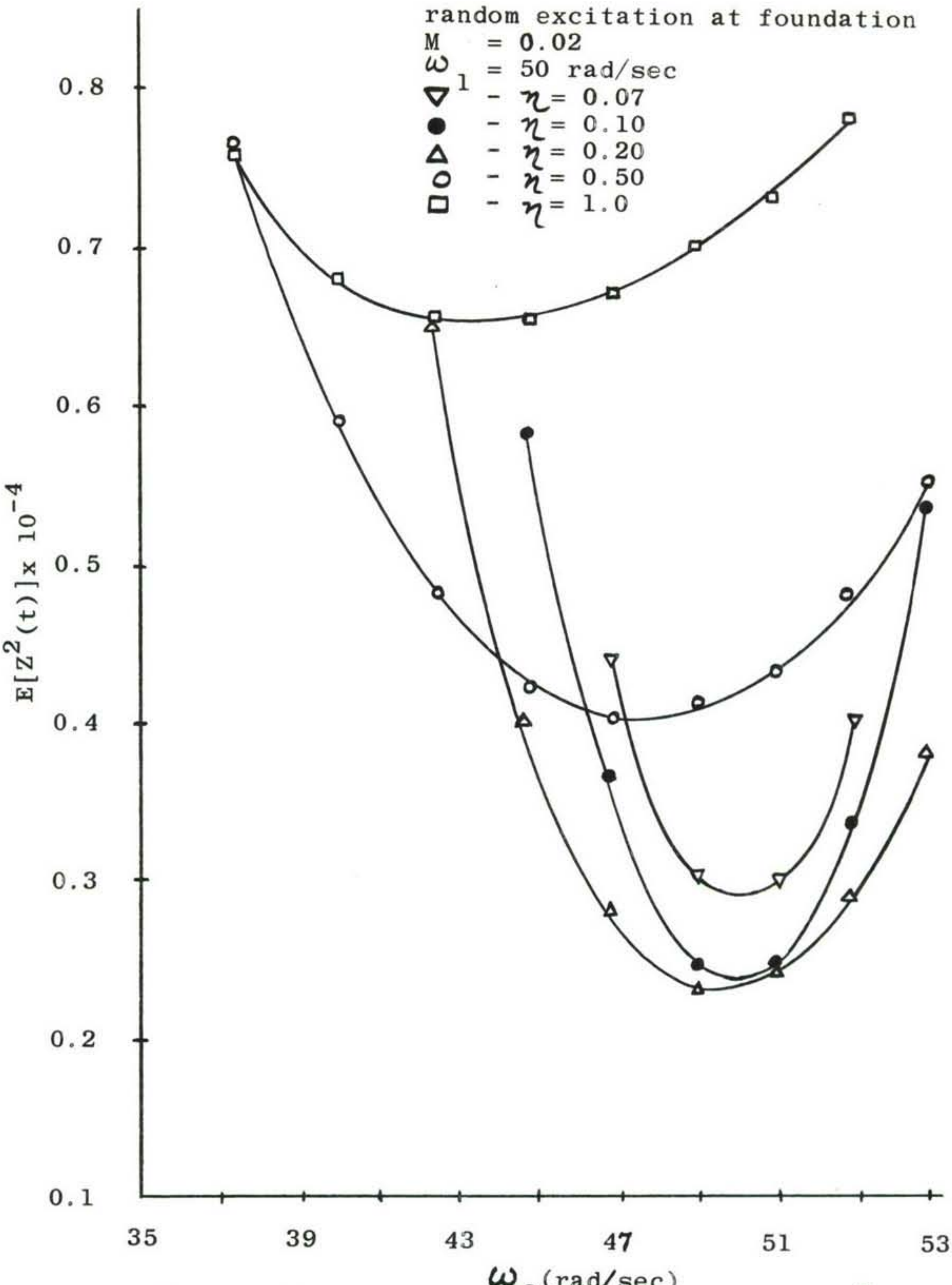

Figure 12. Plot of mean square response $E[Z^2(t)]$ versus natural frequency of damper

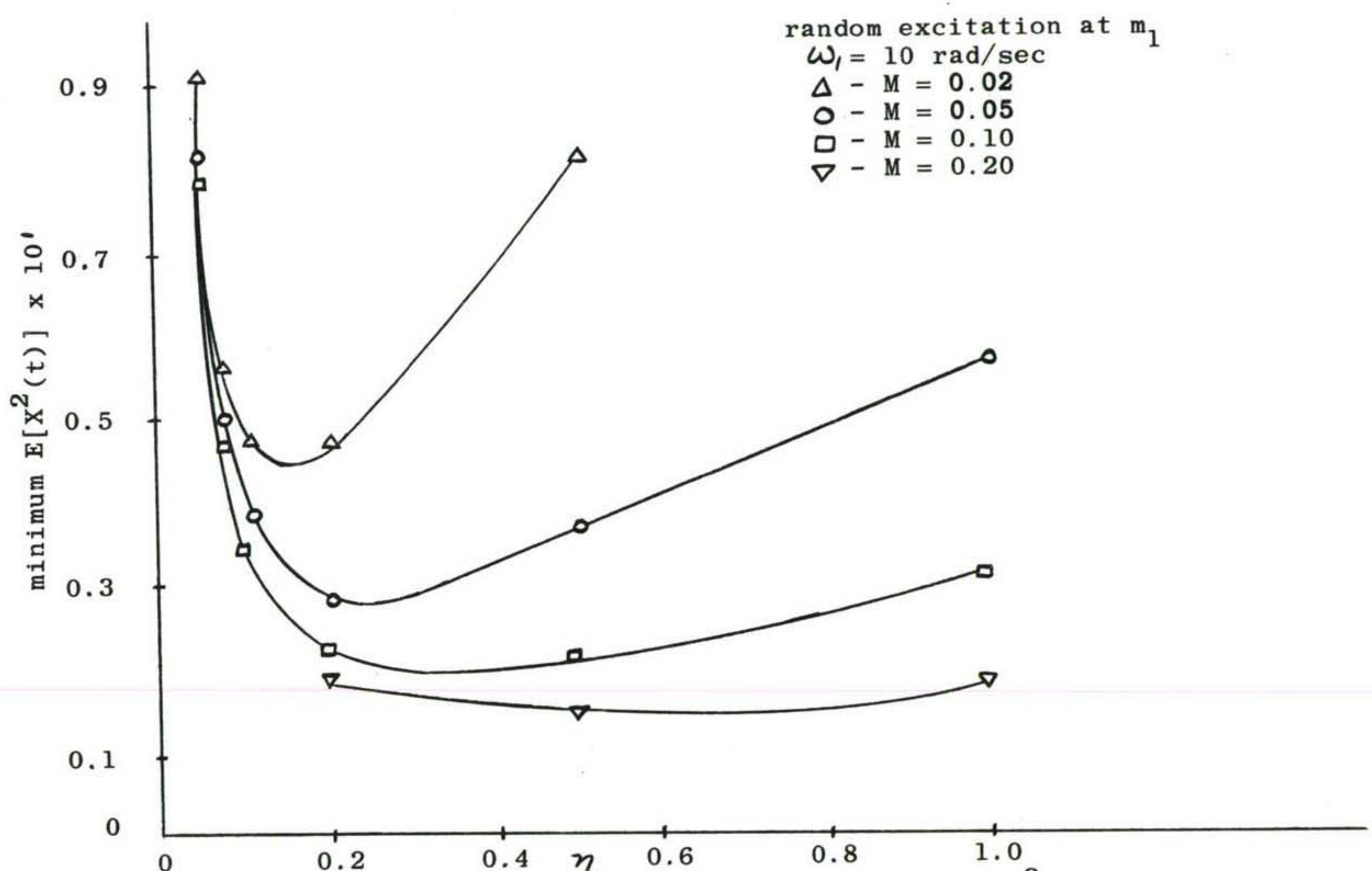

Figure 13. Plot of minimum mean square response $E[X^2(t)]$ versus loss factor η

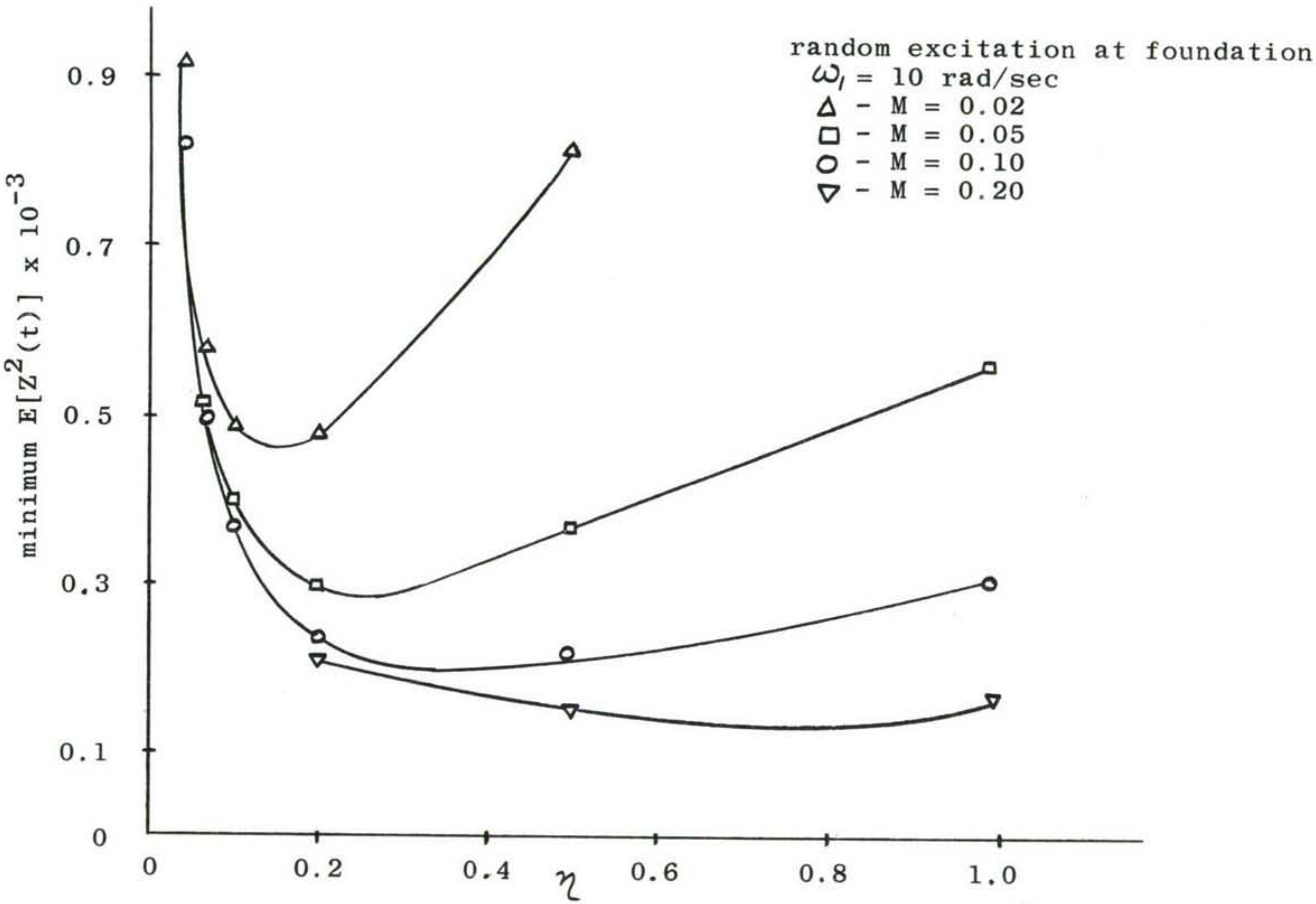

Figure 14. Plot of minimum mean square response $E[Z^2(t)]$ versus loss factor η

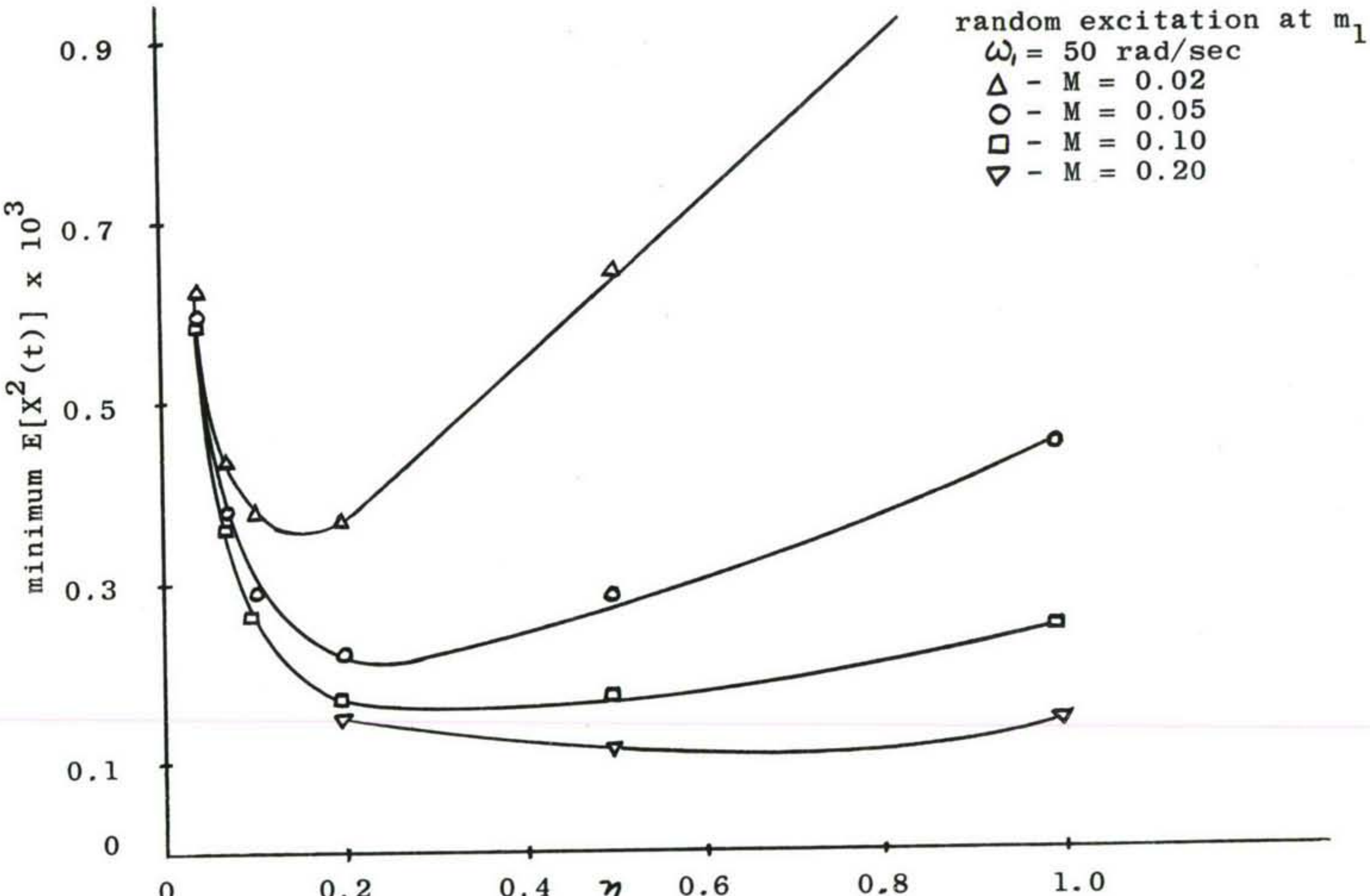

Figure 15. Plot of minimum mean square response $E[X^2(t)]$ versus loss factor η

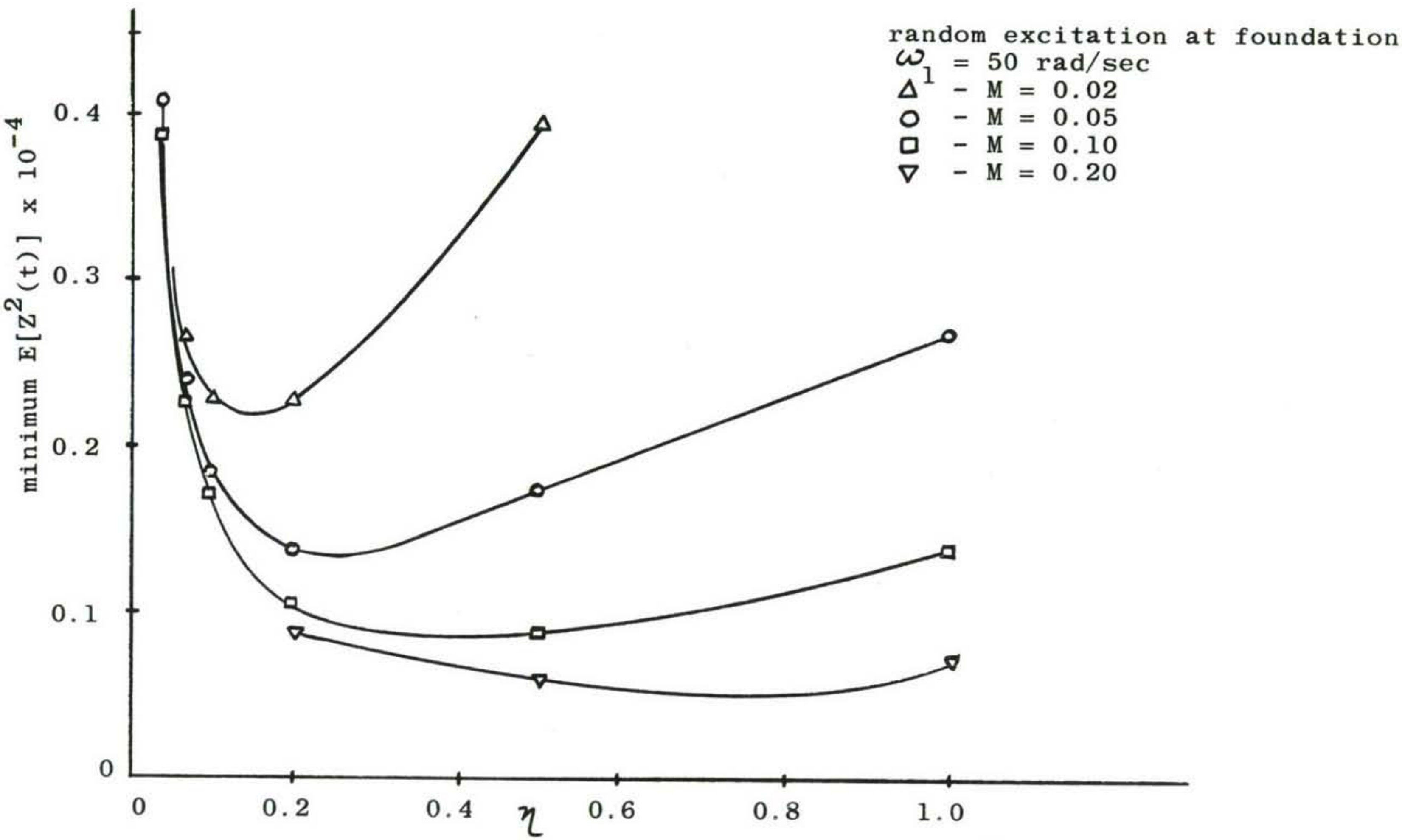

Figure 16. Plot of minimum mean square response $E[Z^2(t)]$ versus loss factor η

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13. ABSTRACT

The objective of this report is to present the effect of a tuned damper on a single degree-of-freedom system which is subjected to white noise excitation. The tuned damper itself consists of a mass connected to a viscoelastic link which, in turn, is connected to the primary system under consideration. The criterion used for tuning the damper is the minimization of the mean square response of the primary system. The tuned damper obtained by use of this criterion is compared to that obtained from another criterion requiring the peaks of the absolute value of the frequency response function to be of equal height.

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